Determination of Electromagnetic Wave Propagation from an Electrically Pulsed Thin Film

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***Abstract*-The Energy Systems Institute is currently performing pulsed-power flashover research on thin films arranged in various geometries. Currently the research is focused on the power transfer characteristics of the film, which consists of a polypropylene layer with a thin aluminum coating on it. It is also known that the surface arcing that occurs when an appropriate electrical pulse is applied to the film is affected by the presence of magnetic and electric fields. By arranging the film in various geometries, the electric and magnetic fields produced by the current will be altered, thereby changing the behavior of the plasma and surface arcing. Therefore, it is important to know the spatial and temporal dependence of these fields so that the resultant surface arcing can be understood and even predicted. In order to do this, a model will be fitted to the experimental current waveform and used in a theoretical treatment of the electrodynamic processes. The necessary calculations will be performed numerically and the results presented as a function of space and time.**

I. Introduction

It is a relatively straightforward matter to determine the electromagnetic fields produced by ideal current carriers such as infinitely long wires or infinite sheet currents. The problem becomes considerably more difficult, however, when the current carrier is limited to a finite spatial extent. Elementary textbook results are no longer valid and one must often resort to numerical techniques based on more general results obtained from Maxwell's equations. The following theoretical treatment will derive a completely general form of the electromagnetic fields, then simplifications will be made to obtain results corresponding to experimental data.

A. Maxwell's Equations and Potentials

To start, the electric field can be expressed in terms of a scalar and a vector potential as follows.

$$\vec{E} = -\vec{\nabla}\Phi - \frac{\partial \vec{\mathbf{A}}}{\partial t} \tag{1}$$

Plugging this into the Maxwell's equations containing source terms, it is possible to obtain differential equations for the potentials in terms of the charge density and current density.

$$\nabla^2\Phi + \frac{\partial}{\partial t}\left(\vec{\nabla}\cdot\vec{A}\right) = -\frac{\rho}{\varepsilon_0} \tag{2}$$

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} - \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\Phi}{\partial t}\right) = -\mu_0\vec{J} \tag{3}$$

Solving these differential equations becomes much easier if one takes advantage of the fact that potentials are not unique. This means that extra terms can be added to the potentials without changing the form of any physical results, mainly the electromagnetic fields. The particular choice that will be used here is called the Lorenz gauge, in which the differential equations involving the potentials reduce to

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{\rho}{\varepsilon_0} \tag{4}$$

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{J}. \tag{5}$$

These differential equations are much easier to deal with since they are now decoupled and are of similar form. The corresponding solutions to (4) and (5) are

$$\Phi(\vec{x},t) = \frac{1}{4\pi\varepsilon_0}\int d^3x' \frac{\rho(\vec{x}',t-|\vec{x}-\vec{x}'|/c)}{|\vec{x}-\vec{x}'|} \tag{6}$$

$$\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi}\int d^3x' \frac{\vec{J}(\vec{x}',t-|\vec{x}-\vec{x}'|/c)}{|\vec{x}-\vec{x}'|}. \tag{7}$$

For the sake of brevity, let $R = |\vec{x}-\vec{x}'|$ and $t_{ret} = t - R/c$. These potentials can then be inserted in (1) to obtain the general form of the electric field for an arbitrary charge distribution and current density. Before doing so, however, it should be noticed that the scalar potential vanishes as long as there is no net charge density on the current carrier. For the purposes of the experiments that these results will be applied to, the current carrier is indeed a neutral metal. Therefore, the net charge density is zero, the scalar potential vanishes, and the form of the electric field reduces to

$$\vec{E}(\vec{x},t) = -\frac{1}{4\pi\varepsilon_0}\int d^3x' \frac{1}{c^2R}\left[\frac{\partial\vec{J}(\vec{x}',t')}{\partial t'}\right]_{ret}. \tag{8}$$

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This is the form of the electric field that will be used in all calculations henceforth [1].

B. Assumptions and Simplifications

The particulars of the experiments carried out by the Energy Systems Institute warrant some simplifications to (8). The experimental procedure can generally be described as follows.

1) A metallized polypropylene film (MPF) sample is cut to a specified size. The metallization is about 100 angstroms thick and the polypropylene provides mostly structural support.

2) The leads of a pulsed power source with an operating voltage of 2.5 kV are placed at each end of the film sample.

3) The source discharges across the film and an oscilloscope displays the current and voltage profiles measured by the respective probes.

Based on the geometry of the film sample and the shape of the current profile, the electric field at any point in space and time can be calculated using (8). In order to do this, the following simplifications will be made.

- The retarded time t_{ret} is approximately equal to the time t at which the field is being evaluated. This assumption is valid because using the retarded time only has significant results when the observation point is very far from the source or when the time at which the field is evaluated is very small. At points far from the source, the fields rapidly diminish to zero. This assumption only yields slightly incorrect results when t is on the order of nanoseconds. As an example, at distances on the order of tens of centimeters, the value of R/c is about 0.3 ns.
- The current distribution is approximately a uniform two-dimensional current on the surface of the film. This assumption is valid because the thickness of the metallization is so small.
- The current profile can be approximated by a step function at $t = 0$ followed by an exponential decay with time. This is true for many, but not all, of the experimental results. An example is shown in Figure 1. This assumption is only preliminary and will be remedied in future work.
- The potentials only go to zero at infinite distances. Due to the rapid decay of the magnitude of the electric field as one moves away from the source, this is a good approximation. However, if a ground plane were to be placed in the vicinity of the current carrier, the results here would have to be modified.

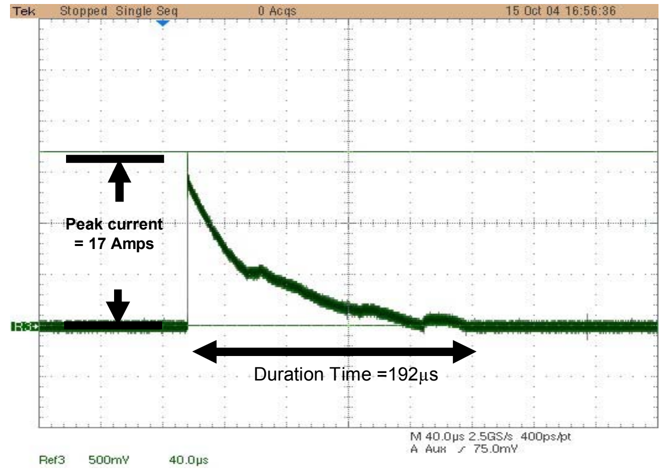

Fig. 1. Experimental results of a test done on a planar MPF sample of width 1.9 cm and length 30.5 cm.

II. Field Calculation

Due to the assumption that $t_{ret} = t$, the current density can be divided into a spatial part and a temporal part. Since the temporal part has no spatial dependency, it comes out of the integral. The two-dimensional integral can be reduced to a one-dimensional integral by hand, but the rest must be performed numerically.

A computer program using the method of Riemann sums was written to perform the calculation. In order to do this, however, certain parameters specific to the experiment must be input. These parameters are the width of the film, the length of the film, the peak current, and the duration of the current pulse. The program then prompts the user to specify the space-time coordinate of interest based on a predetermined axis configuration. In addition, the program calculates the magnitude of the field at a series of points within a specified distance in all directions. The result is that it is possible to view the field strength as a function of all three spatial dimensions plus time. As it is not possible to display the field strength as a function of four variables on a single plot, six separate data files are created that enable the generation of contour plots using any two-variable combinations of x, y, z, and t. Figures 3 through 8 are example plots. The input specifications were as follows: film width = 1.9 cm (0.75 in), film length = 30.5 cm (12 in), peak current = 17 A, and current duration = 192 µs. The point at which the field was to be evaluated was 1 cm above the center of the film, and the range of field evaluation was 0.25 m in all three directions. The time at which the field was evaluated was 140 µs, and the time range was 150 µs in either direction. The origin of the coordinate system is at the corner of the film on the side where the current is coming from, with the x-axis along the width of the film and the y-axis along the length of the film. This is shown pictorially in Figure 2.

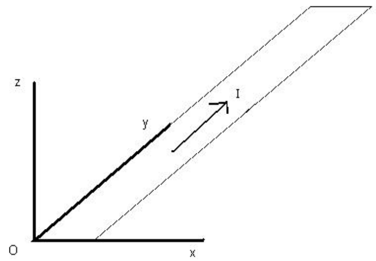

Fig. 2. Film with coordinate axes.

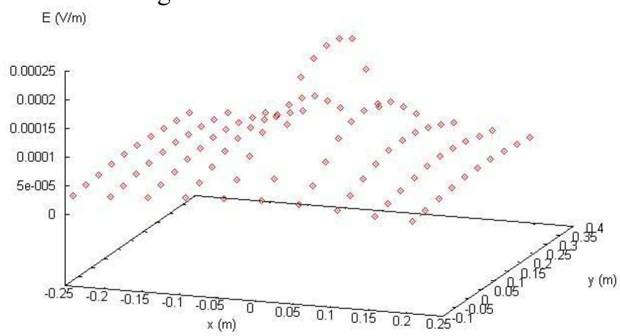

Fig. 3. Field strength as a function of x and y.

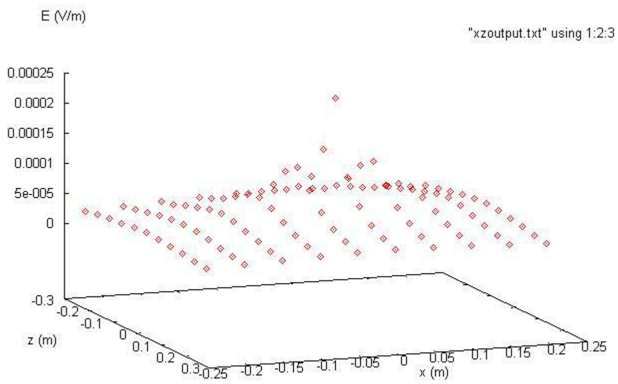

Fig. 4. Field strength as a function of x and z.

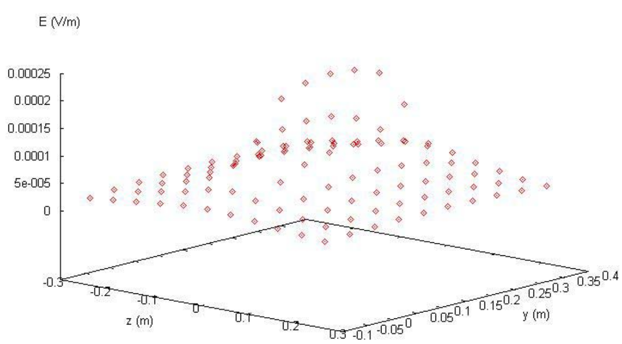

Fig. 5. Field strength as a function of y and z.

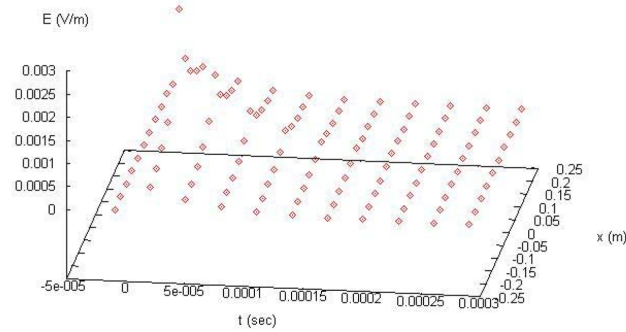

Fig. 6. Field strength as a function of x and t.

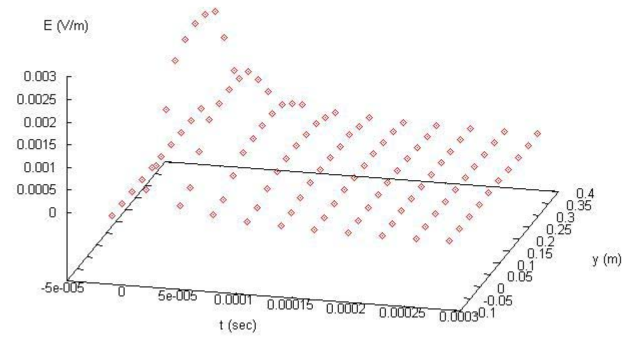

Fig. 7. Field strength as a function of y and t.

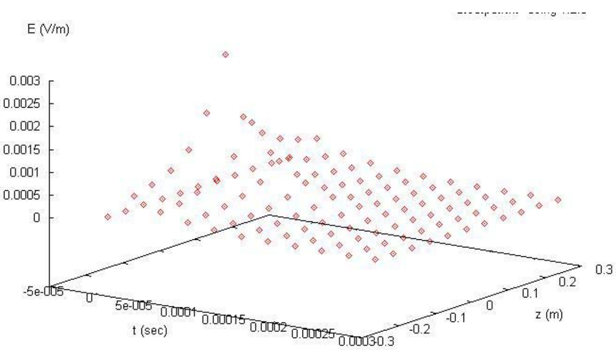

Fig. 8. Field strength as a function of z and t.

It is apparent from these plots that the field strength rapidly diminishes as one moves out from the source. This further justifies the approximation that $t_{ret} = t$. Also, as expected, the maximum field strength of about 3 mV/m occurs at the current peak.

III. Conclusions

Knowing the strength of electromagnetic fields is very important with regard to electromagnetic interference and the potential to create or sustain plasmas. Most theoretical field calculations utilize ideal models and/or assumptions of harmonic time dependence of the sources. The work done here provides a way to calculate the fields produced by any localized two-dimensional sheet current with an exponentially decaying time dependency. The next step is to generalize the time-dependency of the current so that any current data file can be read into the program. This, for example, would help take into account the slight divergences from a pure exponential decay that are characteristic of many of the experimental current profiles obtained by the Energy Systems Institute. An additional future endeavor is to eliminate the assumption of a uniform current density. This would require placing current probes at multiple points on the film and creating a model that accurately reflects the results. Also, additional film geometries, such as cylindrical, could be incorporated. Lastly, in the future, the program will accommodate the inclusion of ground planes or other boundary conditions on the potentials. All of this future work will remedy some of the assumptions, as well as make the procedure for determining field strength applicable to almost any localized current distribution.

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